R E P O R T R E S U M E S

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IN-SERVICE EDUCATION IN ELEMENTARY SCHOOL MATHEMATICS.

NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS INC.

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A BULLETIN WHICH EMPHASIZES THE NEED FOR EFFECTIVE INSERVICE EDUCATION IN ELEMENTARY SCHOOL MATHEMATICS BEGINS WITH AN INTRODUCTORY SECTION WHICH (1) DESCRIBES TYPICAL ELEMENTARY SCHOOL ORGANIZATIONAL PATTERNS AND MATHEMATICS PROGRAMS, (2) LISTS FACTORS WHICH INFLUENCE TEACHERS' ATTITUDES TOWARDS MATHEMATICS TEACHING, AND (3) CITES IMPLICATIONS OF (1) AND (2) FOR INSERVICE EDUCATION PROGRAMS. THE CENTRAL PORTION OF THE BULLETIN DESCRIBES 3 TYPES OF EXISTING INSERVICE EDUCATION PROGRAMS. THE FIRST TYPE IS SELF-DIRECTED STUDY. PUBLICATIONS AND MATERIALS SUITABLE FOR USE IN PROGRAMS OF INDEPENDENT STUDY ARE ENUMERATED. THE SECOND TYPE CONSISTS OF DIRECTED SHORT-TERM STUDIES WHICH ARE CLASSIFIED AS ORIENTATION CONFERENCES AND AS WORKSHOPS AND INSTITUTES. REPRESENTATIVE PROGRAMS OF THIS TYPE ARE DESCRIBED. THE THIRD TYPE IS DIRECTED LONG-TERM STUDY. DESCRIPTIONS ARE PRESENTED OF PROGRAMS OF THIS TYPE WHICH ARE CURRENTLY IN OPERATION UNDER (1) LOCAL SPONSORSHIP, (2) JOINT STATE AND LOCAL SPONSORSHIP, AND (3) JOINT NATIONAL, STATE, AND LOCAL SPONSORSHIP. THE THIRD PRINCIPAL SECTION OF THE BULLETIN DISCUSSES THE ESTABLISHMENT OF GOALS, STANDARDS, AND EVALUATION CRITERIA FOR INSERVICE EDUCATION PROGRAMS AND SUGGESTS WAYS IN WHICH SCHOOL SYSTEMS CAN ENCOURAGE CONTINUING EDUCATION FOR TEACHERS. APPENDIXES INCLUDE A BIBLIOGRAPHY AND SOURCES OF MATHEMATICS INFORMATION. THIS DOCUMENT IS AVAILABLE FROM THE NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS, 1201 SIXTEENTH STREET, N.W., WASHINGTON, D.C. 20036. (JS)

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THE NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS

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IN-SERVICE EDUCATION in Elementary School Mathematics

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Foreword

THE TASK of making in-service education in mathematics available to over a million elementary school teachers presents a sizable challenge. The task involves helping teachers raise their level of competence in mathematics as a body of knowledge, improving methods of teaching, and requiring recognition and analysis of factors that influence children's learning of the subject in the classroom.

OBJECTIVES OF THE PUBLICATION

This bulletin has several objectives:

To provide resource assistance to those involved in conducting in-service education courses for elementary school teachers

To disseminate information to school personnel, and to others interested in the education of elementary school pupils, on the range of in-service activities in mathematics that are currently taking place over the nation

To stimulate increased effort in order to meet more adequately the great need for in-service education that still exists

To help determine the present status and appropriate next steps in mathematics in-service education within a school system

USE OF THE PUBLICATION

The bulletin is designed to be of use to the following professionals:

In-service instructors drawn from the secondary school or college level who have an adequate mathematics background but who lack an orientation to the elementary school program and to the mathematics background of the elementary school teacher

Elementary school personnel responsible for organizing or conducting in-service education programs for their teachers

Foreword

Elementary school classroom teachers seeking improvement through self-directed study

HISTORY OF THE PUBLICATION

This booklet was conceived and developed by the In-Service Education Committee of the National Council of Teachers of Mathematics. The following persons served on the committee during the time of its preparation and contributed to it:

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Agnes Y. Rickey, *Chairman*
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I

The Environment for Mathematics In-Service Education

IN THIS publication the elementary school is being defined as kindergarten through the sixth grade, although most of the material included will apply equally to Grades 7 and 8 in certain organizational structures. Included in this section is selected information about the elementary school that the committee considers essential background for leaders of mathematics in-service programs for elementary school teachers.

THE ELEMENTARY SCHOOL

General Information

Although there is no single typical elementary school, the following observations provide an overview of elementary schools:

1. Elementary schools vary in size from one-, two-, and three-teacher schools to those having more than twenty teachers.
2. Elementary school students generally spend five or six hours per day in school. The length of the school day is often shorter for children of Grades K–3.
3. Cities, counties, and states often specify a required number of minutes per week or per day for mathematics instruction. Some school systems allot as little as fifteen minutes per day in Grades 1–3, while others allot as much as thirty-five minutes. The time allotment for Grades 4–6 seldom exceeds forty-five minutes per day.
4. Curricula of elementary schools consist of several subject fields: language arts, social sciences, mathematics, science, music, art, health, and physical education. The principal and his staff seek to achieve an appropriate balance among the instructional areas of the curriculum.

5. Consultant help in mathematics usually is provided by a general supervisor, although there is a trend toward employing special supervisors in mathematics, either for Grades K–12 or for the elementary school segment of the program.

Organization

Plans of organization in the elementary school vary widely. The following patterns are common:

1. More than 75 percent of the elementary school teachers throughout the country operate self-contained classrooms, where one teacher is responsible for all subject areas for approximately thirty pupils.
2. Schools not organized on the plan of the self-contained classroom usually have some form of partial departmentalization (Grades 4–6), team teaching, multigrade grouping, or nongraded plans (Grades K–3). Such organizational patterns are finding increased use.
3. Some school systems place children in classrooms in heterogeneous groups; others attempt more homogeneous groups on the basis of selected factors, such as levels of ability and reading achievement. Some schools study each child, using a cluster of factors that includes emotional and social development. Usually mathematical ability is *not* used as a grouping factor.
4. Grouping for instruction within the classroom is practiced more in reading than in mathematics and more in early than in later grades of the elementary school.

THE ELEMENTARY SCHOOL TEACHER

It has been stated repeatedly that the teacher is the key to improvement in mathematics learning for boys and girls. Therefore, it becomes important to know some of the factors that influence the teacher's attitude toward mathematics as a subject and mathematics instruction in the classroom. Some of the pertinent problems and conditions follow:

1. Teachers of Grades K–2 have become aware that the majority of children in their classes should have more interesting and challenging content.
2. Innovations in mathematics instruction at the junior high school level have stimulated elementary school teachers to learn about the changes and their implications for the elementary school mathematics program.
3. Breadth of background in arithmetic, algebra, and geometry is a de-

sirable goal for the majority of elementary school teachers. At present, depth study can be expected only for those teachers who anticipate departmentalized or team teaching or who expect to become mathematics consultants. A few states even require each elementary school teacher to have an academic major, which in some instances will be mathematics.

4. Elementary school teachers prefer in-service offerings that combine content and methods of instruction.
5. Some teachers have taught the same grade for many years and lack familiarity with the total elementary school mathematics program; others have had experience in several grades.
6. Generally, elementary school teachers lack confidence in their ability to learn mathematics. This condition may be due to factors such as lack of required mathematics courses in undergraduate work and unfortunate personal experiences in learning mathematics.
7. Because of a lack of mathematics background, elementary school teachers are unduly dependent on the mathematics textbook for the mathematics program of the grades that they teach. This is especially true of the teacher of intermediate grades.
8. An in-service program in mathematics is only a part of the total in-service program for the elementary school teacher.

If they are to be effective, leaders of in-service programs need especially to understand the mathematics background of the elementary school teacher.

While there is improvement at the present time, it is probable that the average elementary school teacher with five or more years of experience has had two, and frequently fewer than two, high school mathematics courses, and no college mathematics course. The typical in-service class represents a considerable range, however—from the teacher whose competence is around sixth-grade level to one who has had exposure to mathematics beyond that presented in most in-service courses.

At a conference on the in-service education of mathematics teachers held in Washington, D.C., March 7 and 8, 1963, it was reported that more than half the states required no mathematics courses for certification of elementary school teachers. Perhaps for this reason many colleges and universities have not felt the need to include mathematics courses in their preservice programs for the elementary school teacher.

A recent study of the curricula of a random sample of 213 colleges and universities that prepare teachers showed that 11 percent require no mathematics in preparing teachers for the elementary school, 30 percent require

only one course, and 21 percent of the mathematics departments offer no courses for the prospective elementary school teacher.[1]

At this conference it was pointed out that, while there has been ample opportunity for teachers of mathematics at the secondary level to attend National Science Foundation Institutes, the same opportunity has been extended to only a minimal number of elementary school teachers. The only large-scale national support for the in-service education of elementary school teachers has been the use of matching funds provided under the National Defense Education Act (NDEA) for programs planned by state departments of education and local school systems.

IMPLICATIONS FOR IN-SERVICE EDUCATION OF ELEMENTARY SCHOOL TEACHERS

On the basis of the foregoing information, the following implications for the in-service mathematics education of the elementary school teacher are indicated:

1. The first offerings in mathematics in-service education should be closely related to classroom work. The content can be directly related to concepts the children will be acquiring in the elementary school mathematics program.
2. The extent to which the elementary school teacher has already been teaching for understanding of mathematics may have a bearing on the speed with which newer content can be assimilated. If the teacher's program has mainly emphasized rote learning, there is likely to be more psychological blocking, and newer concepts will be learned at a slower pace.
3. The effective in-service leader should become acquainted with one or more elementary school mathematics programs of the past ten years in order to understand better how to start in-service work on the proper level.
4. The load of in-service education will not be substantially lessened in the near future, even though some states have recently strengthened requirements in mathematics for certification and others are anticipating doing so.
5. To the extent possible, in-service education courses should be presented by methods that can be adapted for use in teachers' own classrooms. The discovery approach, creative types of exercises, and demonstration classes with elementary school students are appropriate examples.

[1] Veryl Schult and Theodore L. Abell, "In-Service Education of Mathematics Teachers," *School Life*, XLV (July 1963), 29–36.

6. Means should be found to reach teachers of one-, two-, and three-teacher schools, who frequently have no opportunity to participate in in-service education.
7. Teachers should be encouraged to recognize the significance of mathematics for understanding other subject areas and to make use of this interrelatedness as a strong factor in motivating both learning and retention of what is learned.
8. The fact should be emphasized that many of the newer developments are extensions of topics that have been taught for years.
9. State and local school systems need qualified supervisors of mathematics with responsibility for mathematics at the elementary school level in order to meet the increasing demands for the in-service education of elementary school teachers.

II

Types of In-Service Education

THE ULTIMATE GOAL of in-service education in mathematics is to assist each teacher to develop the background needed for effective teaching of mathematics in the classroom. The following classifications show increasing structure in form and increasing depth in content.

SELF-DIRECTED STUDY

The elementary school teacher, or an entire school staff, can make a start in learning newer mathematics by taking advantage of opportunities for informal study. Appropriate resource materials are becoming available in increasing numbers. Descriptions and sources of some of the materials follow:

"THE ARITHMETIC TEACHER" [2]

This magazine, planned specifically for teachers of elementary school, carries general articles on newer trends and developments in mathematics; descriptions of specific experimental projects, results of research, in-service education programs, practical classroom procedures, and methods of instruction; and reviews of books and materials.

RECENT YEARBOOKS OF THE NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS

Yearbooks prepared primarily for the elementary school teacher include the twenty-fifth, *Instruction in Arithmetic* (366 pp., 1960); the twenty-seventh, *Enrichment Mathematics for the Grades* (368 pp., 1963); and the twenty-ninth, *Topics in Mathematics for Elementary School Teachers* (384

[2] *The Arithmetic Teacher* is published eight times a year, October through May, by the National Council of Teachers of Mathematics, 1201 Sixteenth St., N.W., Washington, D.C. 20036.

pp., 1964). The material in the twenty-ninth is organized around the following eight topics and is also available as a series of eight separate booklets:

Sets
The Whole Numbers
Numeration Systems for the Whole Numbers
Algorithms for Operations with Whole Numbers
Numbers and Their Factors
The Rational Numbers
Numeration Systems for the Rational Numbers
Number Sentences

Films and Filmstrips for Elementary School Teachers and Children

Several series of films and filmstrips are available for staff study. An annotated list of films and filmstrips is being prepared by the Project for the Improved Use of Newer Educational Media in Elementary School Mathematics, a project of the National Council of Teachers of Mathematics.

Programmed Materials

Many elementary school teachers have gained some understanding of new topics, such as sets, geometry, numeration systems, and equations, by studying programmed materials, some of which are developed primarily for students. Examples of these are the booklets *Introduction to Sets;*[3] *Mathematics Materials for Use in Workshops, Teachers' Meetings, In-Service Programs;*[4] *Developing Mathematical Understanding* (Books I and II);[5] and the "Mathematics Enrichment" series.[6]

Publications from Demonstration Projects

Individual teachers and school staffs have found the examination and study of materials developed originally in experimental centers or demonstration projects to be helpful in learning new content and modern methods. A brief description of the materials available from these projects follows:

The **School Mathematics Study Group** has published workbook-type texts for Grades K–6 and accompanying teachers' manuals that include some

[3] Eugene D. Nichols, Robert Kalin, and E. Henry Garland, *Introduction to Sets: A Programed Unit* (New York: Holt, Rinehart & Winston, 1962).
[4] Published by the Department of Education of Memphis, Tenn.
[5] Leo J. Brueckner *et al.* (New York: Holt, Rinehart & Winston, 1963).
[6] Programs A, B, and C by George Spooner; Programs D and E by Francis J. Mueller and Alice M. Hach (New York: Harcourt, Brace & World, 1963).

topics new to the elementary school mathematics program and new approaches to topics formerly taught. Materials are available from the Yale University Press.

The **Greater Cleveland Mathematics Program's** texts for Grades K–6 and accompanying teachers' manuals stress a structural approach to the teaching of familiar topics. A booklet for teachers, *Topics for the Elementary Teacher,* presents background mathematics basic to the elementary school program. Materials are available from Science Research Associates.

A teachers' text from the **Madison Project,** *Discovery in Mathematics,* provides a supplementary program containing discovery lessons in mathematics for children and supplementary notes for the teacher. Mathematical content includes axiomatic algebra, coordinate geometry, and the study of functions. Although ungraded, the topics are probably more useful for Grades 3–6 than for earlier grades. Materials are available from Addison-Wesley Publishing Company.

The **University of Illinois Arithmetic Project,** in the monograph *Number Lines, Functions, and Fundamental Topics,* provides suggestions for teachers and supplementary activities for introducing newer topics into the elementary school mathematics program. Discovery procedures are emphasized. Available from The Macmillan Company.

The Stanford Projects offer several publications. *Geometry for the Primary Grades* is a series of textbooks and manuals in which geometry is recommended as a means of extending and deepening mathematical experience in the early grades. Simple adaptations of concepts from Euclidean geometry are the vehicle. This supplementary program is available from Holden-Day. In the series *Sets and Numbers,* a regular program for Grades K–6 is built about concepts of sets. Addition and multiplication are presented as the union of sets and subtraction and division as ways of dealing with subsets. Materials are available from L. W. Singer.

The **Minnemast Project** has developed coordinated materials in mathematics and science for the early grades. These materials are undergoing tryouts in eighteen cooperating colleges and universities located in sixteen states representing all sections of the country. Preliminary units of Minnemast curriculum units in mathematics, K–3, may be purchased through the University of Minnesota Press, 2037 University Avenue, S.E., Minneapolis, Minnesota. The Minnemast Project has prepared a series of twenty lectures and demonstrations for elementary school teachers on the psychology of elementary mathematical concepts. These are available for rental or purchase from Audio-Visual Extension Service, University of Minnesota, Minneapolis, Minnesota.

Publications from State and Local Organizations

Newsletters and bulletins are sent out periodically by many state and local mathematics supervisors and state and regional organizations of the National Council of Teachers of Mathematics. Articles appear on different topics related to the newer mathematics. Among them are discussions of in-service education, briefs of lectures presented at national and regional meetings, and explanations of distinctive features of experimental projects.

DIRECTED SHORT-TERM STUDY

Short-term studies, as brief as one-half to three days and as long as two weeks, are frequently used as a type of in-service education. These may be classified as orientation conferences or as short-term workshops or institutes. Sponsors include local districts, state agencies, universities and colleges, commercial publishers, local associations of mathematics educators, and the National Council of Teachers of Mathematics.

Orientation Conferences

Half-day and full-day orientation conferences have been held to acquaint elementary school teachers with the changes taking place in the elementary school program, to help them understand the nature of changes in both content and method, to interest them in updating programs in elementary grades, and to motivate study of the background mathematics needed for successful teaching of newer programs. Different procedures have been used in conducting these conferences, such as—

1. Demonstration classes, used as a means of orienting teachers to the content and methods that characterize some of the new programs.
2. Mathematics consultants to give an overview or to discuss particular topics and programs.
3. Tape recordings on significant mathematical topics for small school systems that do not have money for consultant service.

In conjunction with the above procedures, short pamphlets or worksheets are desirable. These may be a review of the topics to be presented or structured worksheets for the participants to use during and/or after the conference. In some instances a rather lengthy and well-structured worksheet could be the core of the conference.

Examples of orientation-type conferences follow:

Georgia, under the Superintendent's Curriculum and Instructional Development Program, sponsored mathematics conferences for superintendents

and key teachers in each of three congressional districts. At four all-day conferences held during the year, presentations and discussions were concerned with topics such as the following:

Mathematics in a Changing World
Objectives of a Modern Program
Books, Materials, and Media
Mathematics In-Service Education
Organizing and Administering a Modern Program
The Role of NDEA Title III Funds in the Purchase of Materials
Content and Method in a Modern Program

The responsibilities of both the superintendent and the teacher were considered as participants made plans for similar conferences in their school systems.

During the first year following the release of the Georgia State Guide in Mathematics for elementary teachers, three meetings of two to four hours were held in each of the fifteen regions in the state to create interest in the newer mathematics content, build confidence in teachers' ability to teach the newer mathematics, and stimulate in-service education study groups and courses within school systems.

In **Texas,** teams made up of a mathematics consultant from the Texas Education Agency and an administrator from the local school system held one-day meetings over the state to distribute materials, including a programmed textbook for self-directed study and sets of projectuals for an overhead projector. The morning session was spent in presenting general information on newer mathematics and approaches to teaching. Afternoon sessions of one-and-a-half hours were devoted to sectional meetings. The series of meetings was sponsored by NDEA Title III funds and by the Texas Education Agency.

The **Idaho** State Supervisor of Mathematics enlisted consultants from several sources to hold orientation conferences in sufficient number to be within reach of all elementary school teachers in the state. These were section meetings held in connection with district meetings of the fall State Teachers' Convention.

The **NCTM In-Service Education Committee** has a project under way to hold one or more orientation conferences on elementary school mathematics in sparsely populated areas. From the pilot conferences, guidelines will be developed for the use of others.

Personnel from the U.S. Office of Education have assisted in conducting orientation conferences. Outlines and sample items from work sheets have

been used as the basis for two or more sessions with teachers of primary and intermediate grades who were beginning a study of the more recent trends in elementary school mathematics.[7] Newer approaches to basic topics of regular mathematics programs and some of the concepts of newer programs were highlighted.

Many **publishers** of mathematics textbooks have conducted, or cooperated in conducting, orientation conferences in mathematics for administrators, elementary school teachers, and prospective in-service education leaders. In some cases, a registration fee has been charged to cover expenses.

Of particular importance, because of their availability to many teachers, are the conferences or meetings conducted by local, state, and national **mathematics organizations.** At these meetings there is a variety of presentations, including lectures, panels, discussion groups, and demonstration classes. These differ slightly from the rather narrowly structured orientation conference in that the topics presented at the meetings usually cover the entire spectrum of elementary and secondary school mathematics teaching. Meetings such as these last from one day to three days. Sometimes a nominal registration fee is charged.

The National Council of Teachers of Mathematics and its Affiliated Groups have been most active in this area. The NCTM conducts a national meeting each spring and several regional meetings, each of which serves a several-state area. The NCTM, through its Committee on Affiliated Groups, has supported state mathematics organizations in conducting meetings that serve an entire state or some portion of a state.

Short-Term Workshops and Institutes

These workshops differ from the orientation conference in that they are more highly structured and cover a longer period of time. In general, the programs described here are noncredit programs, but a certificate of participation issued to each member is recognized by the school system as evidence of professional growth for which professional credit may be given. Specific examples of short-term workshops follow:

The **Cincinnati, Ohio,** public school system sponsored a leadership conference for principals and supervisors immediately following the close of the school year. The purpose of the conference was to determine the direction mathematics should take. Emphasis was placed on mathematical strands to be developed in the elementary grades and on teaching techniques. Offerings included addresses by visiting personnel, with implications drawn by local school supervisors responsible for the mathematics program; a panel

[7] Work sheets developed by Edwina Deans and Veryl Schult, mathematics specialists, U.S. Office of Education, for use in short-term workshops and institutes.

of principals on current developments and future plans; and clinic sessions to explore specific problems in greater depth.

Over sixty schools took part in the **Fresno County, California,** three-day in-service workshop for teachers of Grades K–8 which was held just prior to the beginning of school in the fall of 1964. Except for an introductory session, groups of teachers of Grades K–3, 4–6, and 7–8 met separately. Each group had a mathematics consultant. Many teachers gave classroom demonstrations for small groups of their peers at each of the primary-grade levels. Intermediate-grade teachers received instruction by means of closed-circuit television, with teachers serving as discussion leaders after the presentation by the consultant. Outlines and sets of problems were developed for each of the topics considered.

For two summers a one-week mathematics institute has been held for elementary school personnel in **Connecticut.** The institute is sponsored by the Associated Teachers of Mathematics in Connecticut (ATOMIC), an affiliate of NCTM, and by the state department of education. The institute program consists of general lectures, group discussions, classes on special topics, problem clinics, and demonstrations with children. This institute draws heavily on talent within the state for leadership, using persons from outside the state as guest lecturers. In subsequent summers the institute will be moved to different locations in the state in order to reach more teachers.

The state department of education in **Louisiana** sponsored a three-day conference on the new approach to mathematics for all general supervisors in the state. Addresses by keynote speakers were followed by panels and small group meetings.

The **Maryland** State Department of Education and a local school system jointly sponsored a two-week workshop for supervisors and administrative personnel to acquaint them with new ideas and directions in mathematics education at the elementary level. Research, teaching procedures, and materials of instruction were emphasized. The program featured lectures by leaders from experimental projects, demonstrations with children, and study-discussion groups. The superintendent and four representatives from each district in the state attended.

The two-week elementary mathematics workshop held at the **Pennsylvania** State University by the department of elementary education during the summer of 1963 will serve as an illustration of the short-term workshop offered by universities and colleges, which is available in increasing numbers to elementary school teachers. Offerings included general lectures by visiting professors, films on the newer mathematics, observation of demonstration classes of primary- or intermediate-grade children, and discussion periods and work sessions. Principals and elementary school teachers participated.

DIRECTED LONG-TERM STUDY

Long-term study of one-half a semester, a semester, a year, or even two years is the most beneficial type of in-service education for improving mathematics instruction in the elementary grades. These studies may be conducted under (1) the local system's sponsorship, (2) state and local joint sponsorship, or (3) national, state, and local joint sponsorship. Although these types have many common characteristics, examples of the three categories will be presented separately.

Locally Sponsored Study

Locally sponsored workshops are those that are supported financially and conducted by the local school system. While at times the local system may bring in outside consultants and seek advice from state mathematics supervisors or local college or university personnel, the responsibility for developing the program, planning and organizing the in-service work, and following through is accepted by the school system.

The following examples of programs illustrate the variety of ways in which in-service needs have been met by school systems of different sizes and types, such as a small rural town, a large city, a suburban area, and a county system. In general, the purpose of these programs was to prepare teachers to teach newer mathematics by increasing their mathematics background and improving their methods of instruction.

In Greenwood, Delaware, a **rural district** of approximately ten elementary school teachers, a secondary school mathematics teacher attended a two-week summer training session conducted by the state supervisor of mathematics. The training session was directed to demonstrating typical presentations with elementary school teachers and familiarizing the prospective leaders with elementary textbooks and programs. The secondary school teacher, in turn, presented fifteen weekly two-hour sessions after school. NDEA Title III funds were used in part to employ the secondary school teacher as a state consultant to lead the in-service program.

In Harrington, Delaware, the district's thirty elementary teachers were given one hour of released time per week and used an hour of their own time to attend a series of fifteen two-hour sessions taught by the state supervisor of mathematics.

In Seaford, Delaware, an elementary teacher with a substantial mathematics background was recommended by the district to attend a special training program for potential leaders that was conducted in a series of summer sessions held at the University of Delaware. He was later employed as leader of a series of eight two-hour seminars to supplement a television in-service series, "Sets and Systems."

The Oxford, Pennsylvania, school system, with seventy-eight teachers in Grades 1–8 (48 of them in Grades 1–5), employed consultants from a neighboring institution, the University of Delaware, for a twenty-hour workshop, consisting of two five-hour sessions held on Professional Days and five sessions of two hours each, one hour during school time and one hour after school. The workshop was followed by a series of five one-hour presentations by textbook representatives and a textbook selection by the faculty.

District administrators felt the workshop provided a good overview for the faculty and took away apprehension among the teachers about the changing curriculum, but also showed a need for strong leadership in the changeover. A classroom supervisor of mathematics was subsequently employed to provide this leadership.

In Newport, Delaware, one district sponsored a two-week summer workshop which two teachers from each grade were paid to attend. During the workshop, teachers were introduced to the mathematics involved through the Colonial filmstrips designed for teaching children. They also viewed the SRA films designed for in-service education of elementary school teachers. Finally, the workshop participants developed written materials to point out similarities and differences between the existing program and the emerging program. The materials were designed to assist teachers not in attendance in the summer session during the transition period in teaching children.

A new series of textbooks was introduced the following year in Grades 1 and 2. The leader of the workshop was a local elementary school teacher who had taken mathematics courses designed for elementary school teachers and who had made a study of elementary textbook series. A subsequent series of after-school sessions was conducted to familiarize the remaining teachers in Grades 1 and 2 with the content of the films and filmstrips.

A year later an additional in-service series of fifteen two-hour sessions was conducted immediately after school hours by a mathematics supervisor from the State Department of Public Instruction. The series was designed to implement further a planned use of new textbooks in higher grades.

Cincinnati, Ohio, a **large city school system,** uses a highly structured and diversified in-service education program to implement change in elementary school mathematics. Salient features of the program are summarized below.

A four-day institute is held each fall during the week before the opening of school. At this time, outside consultants assist in such areas as mathematical background, enrichment, remedial work, and methods and materials of instruction. Teachers' participation is voluntary.

A one-week workshop is held in June immediately following the close of school. This is directed by a visiting consultant approved by the University

of Cincinnati, which offers academic credit for the workshop. Members of the local supervisory staff assist the consultant. Emphasis has been given to challenging the most able children in primary- and intermediate-grade classes.

A two-day workshop is held near the midyear for teachers participating in the experimental accelerated program. A consultant from outside the local system is used. Substitutes are hired for the participating teachers during the period of the workshop.

During the course of the year certain individual schools or groups of schools provide workshops as specific needs are identified by teachers, principals, or supervisors. During the current year, one of these workshops includes the principals and teachers of six schools within a particular geographical area.

Guided visiting days are planned, during which teachers see demonstration lessons in the morning and participate in follow-up discussions in the afternoon.

First-year teachers participate in regularly scheduled practicum sessions, with local supervisors serving as consultants.

After-school telecasts are provided, dealing with such topics as number-line games, arithmetic with frames, physical representations as an aid to abstraction, use of textual materials, and providing for individual differences.

Programs of the Mathematics Club of Greater Cincinnati are designed particularly for elementary school teachers.

Special committees of teachers and others deal with such problems as establishing criteria and selecting basic texts, supplementary books, films, filmstrips, and other visual aids; developing the arithmetic section of curriculum bulletins for primary and intermediate grades; and preparing bulletins relating to low achievers and to able learners. "Arithmetic Enrichment Ideas for Grades 1, 2, 3" and "Enrichment Ideas for Grades 4, 5, 6" were developed as an outgrowth of a June workshop.

Enrichment telecasts for children have been prepared on topics such as "What Is a Set?" "Kinds of Sets," "What Is a Subset?" "What Are the Uses of Zero?" "Let's Look for Patterns," "Union of Sets," "Intersection of Sets," "Prime and Composite Numbers," "Different Names for the Same Number."

The city of Los Angeles has an in-service education plan directed toward the introduction of new textbooks into the system within one year. A series of study booklets for workshops to be held in individual schools is being prepared by an Elementary Mathematics, K–6, Committee to implement the long-range in-service plan of the city. Text materials in the adopted series will also be used as source materials. The first booklet presents an over-

view of newer mathematics for the elementary school. Two ways of implementing modern ideas in the classroom and suggested: (1) placing more emphasis on the structure of mathematics, and (2) bringing a more dynamic teaching approach to the subject.[8] The second booklet presents discussions of number systems, numeration systems, teaching techniques, and mathematical terminology.[9] These are two of several booklets on modern ideas in elementary school mathematics to be prepared.

Suggestions in these first two booklets are limited to those that do not require extensive in-service training. It is expected that the newer mathematics presented in these bulletins will be fused with the regular program during the year of study prior to the introduction of new textbooks into the system. Such topics as sets, mathematical sentences, geometry, and logic, which require more teacher background, will be developed in later bulletins. Workshops in the schools will be supplemented by a TV program. Later phases of the plan include study of the text materials selected and demonstration lessons.

The Phoenix, Arizona, public school system offered in-service education for teachers of Grades 1–8 that was planned, organized, and directed entirely by local school personnel. The purpose of the course was to provide teachers with an opportunity to become acquainted with the main themes in contemporary mathematics. Eight sections were set up, one for teachers of each grade. Approximately three out of four elementary school teachers in the system participated in the program.

Three hours' professional-growth credit was given to those teachers who met the requirements, which included attending at least sixteen sessions of two hours and forty minutes each and responsibility for homework assignments and tests administered.

Brief surveys of the following areas provided the content for the course.[10]

Numeration

Review of ancient systems of numeration and investigation of other number bases

Sets and Set Notation

A look at one of the most powerful ideas of modern mathematics, with emphasis on definition of set, operations with sets, and notation

[8] *Implementing Modern Ideas in Elementary School Mathematics*, Part I ("Overview Instructional Bulletins," No. EC-79 [Los Angeles, Calif.: Los Angeles City Schools, Division of Instructional Services, 1964]).

[9] *Interpreting Modern Ideas in Elementary School Mathematics* ("Overview Instructional Bulletins," No. EC-78 [Los Angeles, Calif.: Los Angeles City Schools, Division of Instructional Services, 1964]).

[10] Information provided by C. B. Smith, director of curriculum, Scottsdale Public Schools, Phoenix, Arizona.

Frame Arithmetic
Equations, inequalities, and open sentences presented through the use of frames

Structure of Mathematics
Closure, commutative, associative, distributive, identity, and inverse properties studied in relation to the structure of groups, rings, and fields

The Number Line and Coordinate Axis
Fundamental operations taught through the use of the number line; graphing on the coordinate axis

The Integers
Operations and properties of the integers; primes and factoring

The Rational Numbers
Discussion of addition, multiplication, greatest common factor, lowest common denominator, and changing to simplest form

Geometry and Measurement
Investigation of properties of simple geometric figures; discussion of the arbitrary basis for measurement

Ratio and Percent
The concept of percent taught through the use of ratio

Optional Topics
Logic; probability

Bucks County, Pennsylvania, a **county school system** of thirteen school districts and over a hundred elementary school teachers, saw the basic problem in introducing a new mathematical program as a matter of changing teachers' attitudes.[11] Toward this goal there was much active involvement of the teachers in activities such as the following: meetings with administrators of the thirteen districts; two-week workshops for teachers, with large-group lectures in the morning and small-group work in the afternoon to discuss manipulative and visual materials and to prepare classroom materials; in-service workshops during the regular school year; and classroom demonstrations and informal discussions. During the process, leadership was given by supervisors and master teachers. Those who worked in the program concluded that changes take place when—

1. Workshop supervisors plan for activities with those who participate.
2. It is determined which factors hinder success and which facilitate it.
3. Open communication lines are developed and maintained between teachers and leadership personnel.

[11] Betty Atwell Wright, "Anatomy of Change in Elementary School Mathematics," *The Arithmetic Teacher*, X (March 1963), 158-61.

4. Opportunities are provided for explaining the program to parents and other lay persons.
5. Provision is made for continuous mutual support and understanding between teachers and leadership personnel.
6. Procedures for evaluation and feedback are developed.
7. Teachers are helped to become more self-directed.
8. Local districts provide teachers supportive services.

Dade County, Florida, which contains Miami and its suburbs, has around 3,800 teachers serving 107,000 elementary school children in approximately 150 school centers.[12] The county is divided into six districts. A consulting mathematics teacher is assigned to each district to improve the teaching of mathematics. The elementary school mathematics supervisor and the mathematics consultants plan the program for the county. Every other week a consultant spends a full day in each participating school. During the day the consultant teaches demonstration lessons at each grade level. Other teachers of that grade are released to observe the demonstration. An after-school planning session is held with all members of the staff to familiarize the teachers with the content of the program for the coming weeks and to evaluate past progress.

Other in-service activities designed to update the program in Dade County include—

1. Noncredit courses staffed by county personnel with special training in modern mathematics. SMSG Vol. IX is used as a text for these courses.[13]
2. TV demonstrations given twice a week for teachers of Grades 1 and 2, each one accompanied by supplementary materials.
3. A summer TV enrichment program.
4. Special meetings, demonstrations, and bulletins planned especially for principals and assistant principals to inform them about in-service activities and to solicit support for such activities.
5. Curriculum bulletins covering the basic program.

Materials developed by the mathematics supervisors and the consulting teachers include two bulletins designed to enrich the traditional program with newer topics and with more challenging and interesting approaches to

[12] J. Fred Weaver, "Effecting Change in a Large County System," *The Arithmetic Teacher*, X (May 1963), 294–97.

[13] Max S. Bell *et al.*, *A Brief Course in Mathematics for Elementary School Teachers* (School Mathematics Study Group, "Studies in Mathematics," Vol. IX [Stanford, Calif.: Leland Stanford Junior University, 1963]). Distributed by A. C. Vroman, 376 S. Pasadena Ave., Pasadena, California.

familiar topics and a bulletin setting up the sequence of learnings for the development of concepts.[14, 15]

Study Under Joint State and Local Sponsorship[16]

The illustrations of workshops under joint state and local sponsorship demonstrate a close working relationship between the state mathematics department and the local school system, with each sharing certain definite responsibilities in in-service education.

Some of the reports emphasize organizational procedures that facilitate in-service education; others stress program content. Again much variation is in evidence, because states and local systems differ in in-service education needs and the available means of meeting them.

National Defense Education Act Title III funds allocated to states by the federal government on a matching basis may be used to provide supervisory and related services in mathematics. Some states use these funds to pay instructors and to provide materials.

The following detailed plan for organizing noncredit workshops has been widely used in the state of **Georgia.** The guidelines, developed by the state department of education in cooperation with local school systems, may be helpful to others with similar needs.

Purpose.—To increase proficiency through a local workshop for teachers who find it impossible to attend classes on a college campus.

In the workshops, attention is given to increasing the person's knowledge and understanding of content and to overall curriculum and organization for instruction at the level at which he teaches.

Possibilities and Limitations.—Workshops are sponsored by the state department of education in any location where at least seventy-five participants can be enrolled. In almost all instances this means cooperation between several school systems. However, in situations where only one school system is involved, workshops with total enrollments of less than seventy-five participants may be approved.

Workshops will generally be held for two weeks (during the summer only) and will carry no college credit. The workshop day will be six hours, with leaders and participants available the full day to work together as needed in planned activities.

The workshops must be scheduled when leaders can be available. Every

[14] *Enriching Arithmetic,* Primary Grades 1–3, Intermediate Grades 4–6 (Bulletin No. 7A [Miami, Fla.: Dade County Public Schools, 1962]).

[15] *Sequence of Learnings in Arithmetic,* Grades 1–6 (Bulletin 2-A-1a [Miami, Fla.: Dade County Public Schools, 1964]).

[16] Inquiries concerning materials identified by state should be addressed to the mathematics supervisor of the state. (See Appendix B for a listing of needed information by state.)

effort, however, is made to schedule them at times of stated preference of those enrolled.

Responsibilities.—Each organizational level bears several responsibilities in regard to the program, as follows:

State Department of Education
- Coordinate the statewide program.
- Make the staff available for summer workshops.
- Provide additional leaders for local workshops so as to have one staff member for each twelve to fifteen participants.
- Finance and direct a leadership training workshop of one week's duration.
- Issue certificates of participation if requested.
- Plan for statewide evaluation of the project.

Local School Systems
- Designate a local coordinator.
- Survey the local needs and interests.
- Supply the State Department of Education's Workshops Coordinator with information requested.
- Agree to make available necessary materials and supplies.
- Provide adequate working space, clerical help, and overall coordination.
- Reach agreements among the cooperating systems for sharing the financial load.

Local Coordinators (for each cooperating system)
- Plan for and conduct the preliminary survey of local needs for a non-credit workshop.
- Assume leadership in preplanning to identify needs and clarify problems.
- Assume responsibility for presession organization.
- Assume responsibility for roll-keeping and similar duties at workshop sessions.
- Follow up the workshop by assisting participants in implementing plans or applying increased understanding in the classroom.

General Coordinators
- Carry out the responsibilities of local coordinators.
- Assume responsibility for selection of and arrangements for physical facilities.
- Cooperate with the staff in getting materials and supplies in readiness.
- During the workshop, participate fully in all overall planning as a staff member.

Staff Members (present department staff and extras)
- Participate fully in the training period.

Make a list of materials, supplies, and resources deemed necessary for a particular workshop, and either see that the items are on hand, or make these needs known to the general coordinator in time for them to be procured.

Carry out an assigned role in a particular curriculum or problem area and also participate in overall planning.

Attend regularly and participate fully in all activities of the workshop.

Each Participant

Make specific plans during the workshop for putting new life in teaching the following year.

The state of **West Virginia** sponsors a ten-week course in mathematics for elementary school personnel that meets for two hours per week. The lessons, which have been prepared by the state mathematics specialist, provide an overview of the newer thinking in mathematics and background mathematics for the elementary school program.[17] The importance of concepts and their application to the total mathematics program is stressed. Some attention is given to the use of teaching aids.

The extent to which methods become a part of the course depends upon the instructor. The instructors selected have taken the course themselves, as well as having prior mathematics background. Whether secondary school teachers, elementary school supervisors, or key teachers, they have evidenced an interest in elementary school mathematics and an ability to communicate successfully with elementary school teachers.

Lessons provide an introduction to systems of numeration, operations, the rational- and real-number systems, and informal geometry and measurement. Each lesson develops the mathematical background and presents a series of related exercises and a bibliography related to the topic. Liberal use of diagrams and charts helps to organize and clarify ideas.

Request for the course is made by the county superintendent to the state mathematics specialist. A small library from the state department of education is loaned to each center for the duration of the course. The salary of the workshop instructor and the cost of materials come from NDEA Title III funds.

The bulletin *Guidelines for Mathematics—Elementary*,[18] prepared by the state of **Washington,** has been used as basic content for in-service workshops throughout the state. The purposes of the bulletin are to assist teachers and school districts in the analysis of programs and materials and to offer suggestions and plans for in-service training programs. The bulletin

[17] Seaton E. Smith, *Modern Mathematics for Elementary Teachers* (Charleston, W. Va.: State Department of Education, Division of Instruction, 1964).

[18] Robert R. Willson, *Guideline for Mathematics—Elementary* (Olympia, Wash.: State Department of Public Instruction, 1963).

contains a discussion of recent changes, of the goals for elementary school mathematics, and of the underlying ideas in a modern program. An outline of content covers the following major strands of mathematics: concepts of sets, number and numeral, numeration systems, geometry, properties and techniques, equalities and inequalities, measurement, and statistics. Suggested activities illustrate some of the ways in which appropriate concepts of each strand may be developed in each grade of the elementary school. Also included is a list of criteria for textbook selection, discussions of the algebraic structure of the number system, foundations of algebra, and intuitive foundations of geometry; and a list of materials of instruction as basic content for in-service workshops throughout the state.

The following report from a mathematics supervisor of the **Virginia** State Department of Education illustrates the way in which the department met a local school system's request for assistance in making a smooth transition from traditional to contemporary materials. This particular school system had adopted contemporary materials for Grades 1–4 the previous year. The next year the adoption would be extended through fifth grade. The specific purpose of the in-service program was to acquaint the fifth-grade teachers with the materials they would be teaching during the next year.

The following procedures were set up to carry out this request. Eight meetings, one every third week, were held with these teachers to allow sufficient time to achieve the objectives outlined below.

1. To make each participant feel more at ease in handling the fifth-grade materials
2. To establish an increased understanding of the problems of their fellow workers
3. To establish an increased awareness of the developmental processes of the children they teach
4. To acquaint the participants with varying teaching techniques being employed
5. To increase cooperation among the various faculties
6. To acquaint the participants with the mathematical content and development of concepts
7. To assure continual growth of the pupil

During these meetings the content, concepts, and methodology for each grade were discussed. At the initial meeting the following stipulations were made:

1. Each participant was to obtain a text for the grade to be covered at each of the eight sessions (Grade 1 at the second and third meetings,

Grade 2 at the next, and so forth). Textbooks were loaned by the school division.

2. Each participant was to visit *at least* two classes at that level before the next scheduled meeting of the in-service group.
3. Each participant was to meet with the teacher of the class visited to discuss the program.
4. Each participant was instructed to observe the children, their maturity, and their reaction to the materials. It was stressed that the purpose was not to observe the teacher critically but rather to learn.
5. Each of the principals in the elementary schools involved was to arrange for released time for the participants in his school to take part freely in the above activities.

The state department of education of **New York** has prepared a syllabus, *Mathematics K-6—A Proposed Program*, in which the major concepts in six basic areas and content expectations are outlined by grade.[19] Accompanying the syllabus is a series of six small supplementary bulletins, one for each of the six areas developed in the syllabus.[20] The six titles are these:

Number and Numeration
Sets: Language and Symbols
Operations on Whole Numbers
Operations on Fractions
Problem Solving
Geometry and Measurement

In-service programs are made available at regional centers over New York State. There are noncredit sixty-hour courses that meet for two hours each week throughout the school year. The purpose of the in-service program is to give teachers background mathematics in the structure of the number system and the principles underlying the algorithms and to provide them with ideas that can be used directly in the classroom. Teachers test their understanding of the newer topics and approaches by working through several pages of exercises after each topic is presented.

The state of **Rhode Island** assists local communities with in-service education, using a sliding scale of compensation. The first year the state pays two-thirds of the instructor's salary and the community pays one-third. The second year the proportions are reversed, the state paying one-third and the

[19] *Mathematics K-6—A Proposed Program* (Albany, N.Y.: The State Education Department, Bureau of Elementary Curriculum Development, 1963).
[20] *A Series of Information Pamphlets About Elementary School Mathematics* (Albany, N.Y.: The State Education Department, Bureau of Elementary Curriculum Development, 1964).

local system paying two-thirds. The third year the state gives assistance in setting up the program, but the local system bears the total cost.

One of the first state programs of in-service education for elementary school teachers has been in operation in the state of **Illinois.** Interest and desire for the program were sparked through six yearly regional meetings of the Illinois Council of Teachers of Mathematics. First tried out as a pilot project in three centers, the program has grown each year. Teachers from all sections of the state have participated in these noncredit workshops of eight meetings, held once a week in sessions of two hours each. The bulletin *Teacher's Guide for Number and Operation Workshops,* prepared by state supervisors of mathematics, is used by instructors as a basis for the content of the workshop.[21] At each class meeting, participants receive class notes that supplement the instructor's lecture, suggestions for implementing the ideas in the classroom, and exercise and assignment sheets. Simple evaluation forms are filled out by participants at each meeting. This program was conceived with the hope that participation in the workshops would stimulate many teachers to lose their fear of mathematics to such an extent that they would enroll in credit courses, and this has indeed been the case.

The **New York** State Department of Education makes grants available to elementary school teachers for course work in newer mathematics. The purpose of the program is to improve the instructional competence of teachers and to enable them to serve as resource teachers.

Under this plan applicants for summer institutes are recommended by their superintendent or principal. Teachers are eligible who have had at least three years of experience in the classroom and expect to teach in the state the following year. Applications are made to colleges and universities offering state-sponsored programs. Applicants must meet the admission requirements of the host institution.

The state of **Arkansas** has planned an in-service program for elementary school teachers to implement the goals of modern mathematics in all elementary schools, and at least one in-service course for every elementary teacher before the next state adoption of mathematics textbooks.

After a tryout with sixteen schools and over 500 elementary school teachers and revision of materials, an In-Service Mathematics Program for Arkansas Classroom Teachers (IMPACT) is now available for use over the state.[22] It includes the following:

General information sheet describing the program
Application forms

[21] *Teacher's Guide for Number and Operation Workshops in Elementary School Mathematics* (Springfield, Ill.: Office of the Superintendent of Public Instruction, 1960).

[22] Robert M. Todd and Cecil W. McDermott, *Programed In-Service Mathematics Materials* (Little Rock, Ark.: Arkansas State Department of Instruction, 1965).

A booklet, *Instructions for Organizing the In-Service Mathematics Program for Arkansas Classroom Teachers*

Four reference booklets prepared by classroom teachers in cooperation with the state department of education on these topics: sets, numeration bases, mathematics vocabulary, and the number line

Eighteen programmed homework lessons

Group projects

Reference materials

Sample copies of new textbooks

Examination and aptitude tests

The **Colorado** Department of Education has prepared fifteen pamphlets to be used by elementary school teachers in a self-directed study program of fifteen sessions.[23] The content of the pamphlets and film excerpts of classroom demonstrations are adapted from a "packaged" in-service program by the Madison Project.[24] The film excerpts, presenting children learning mathematics, illustrate newer content and the discovery method of teaching.

Pamphlets and film excerpts serve as a basis for group discussion as mathematics and mathematics teaching are studied simultaneously. It is believed that this approach will decrease the lag between the development of newer teaching procedures and their wide application in classroom practice.

The pamphlets and film clips are designed for weekly meetings of approximately one-and-a-half hours. A discussion leader, chosen for each session well in advance of the scheduled meeting, plans and conducts the session. The first pamphlet in the series describes the program and stresses certain features that should be emphasized, such as these: The program is supplementary in nature rather than all-inclusive; it presents some mathematics that can be but has not usually been taught; and it is designed to help teachers develop techniques that will implement the discovery approach.

The University of **California** at Berkeley and the California State Department of Education, with the support of NDEA funds, have developed an in-service program in modern mathematics for elementary school teachers as one means of implementing the "Strands of Mathematical Concepts" of the Advisory Committee on Mathematics to the State Curriculum Commission.[25] "Understanding Rational Numbers," a series of twenty-eight half-hour video tapes, has been prepared for presentation by open-circuit TV.[26]

[23] *Mathematics for the Elementary School—An In-Service Course for Elementary School Teachers* (Denver, Colo.: Colorado Department of Education, 1964).

[24] See pages 29-30 for a description of the Madison Project in-service course.

[25] "Strands of Mathematical Concepts" (reprint from *California Mathematics Council Bulletin*, Vol. XX [Fall 1962]). Published in Sacramento, Calif., by the State Department of Education.

[26] Robert Schute, "Understanding Rational Numbers" (kinescope lecture film series [Sacramento, Calif.: State Department of Education, Bureau of Elementary Education]).

Typical topics presented in the film series are these:

Sets of Numbers in Arithmetic
Properties and Operations I, II
Historical Development of the Hindu-Arabic System
Mathematical Sentences
The Language of Sets

Commentaries on each topic are available for study with the films. The films stress the mathematics content that is considered most basic for teaching elementary school mathematics. Five of the twenty-eight are classroom demonstrations showing children dealing with newer topics in mathematics and responding to approaches leading to the discovery of generalizations.

A unique feature of this in-service program is the plan to gather data on the effectiveness of the program under the following conditions: viewing alone; viewing in a group with no leadership; and viewing in a group with trained leadership. Data were also collected on the understanding of the topics among a control group who did no viewing and received no regular class instruction.

Participating teachers take a pretest and posttest as a part of the requirement for receiving credit. Data on pupils are obtained from standardized achievement tests administered at the beginning and end of the in-service program. Teachers are observed in the classroom by interviewers at the beginning and end of the investigation. The program is offered as a correspondence course of the University of California Extension. A syllabus for the course is available.[27]

In 1965, the state of **Iowa** conducted the Northwest Iowa Telewriter Project, a pilot project on the use of the telewriter as a means of large-scale communication for an in-service program in elementary school mathematics. Programs may originate from any source and be communicated simultaneously by phone circuits to as many "receiving centers" as desired. The conference phone and telewriter enable both oral and written communications to pass from the source to the receiving centers and back to the source if necessary. This medium allows instructors to communicate with many people in many different areas at the same time.

Study Under Joint National, State, and Local Sponsorship

While most financial sponsorship for the in-service programs presented in this section comes from national sources, some of the projects are sponsored jointly by two or more organizations. Specific examples follow:

[27] Arden K. Ruddell and Adrian N. Gentry, *Understanding Rational Numbers* (available from Educational Film Sales, University Extension, University of California, Berkeley, California).

"Patterns in Arithmetic," a television program for pupils of Grades 4, 5, and 6, was supported by the Wisconsin Improvement Program, the National Science Foundation, and the cooperating schools.[28] The purposes of this program were to determine—

1. Whether a new approach to arithmetic at these grade levels could be taught effectively by television.
2. Whether an effective in-service program for the teachers could be provided simultaneously with the television program for children.

Approximately seventy fifteen-minute television lessons are available for each of the grades four, five, and six. The topics covered are these:

Sets
Numbers and operations
Number systems
Numeration systems
Computation
Problem solving
Measurement
Informal geometry
Rate-pairs (ratios)
Elementary number theory

It is recommended that two television lessons be presented each week. Participating classes are provided with exercises and explanations of mathematical concepts. Teachers have an orientation session about every third week so that the mathematical concepts the children will be meeting in the next six lessons can be presented. An occasional enrichment lesson, which is optional, provides challenging material for the able children and makes it possible for the teacher to meet individual differences within the class.

Kinescopes of "Patterns in Arithmetic," workbooks for pupils, and teaching suggestions are available for use by other school systems.

"Sets and Systems," a television series of fifteen lessons of thirty minutes each, is designed to provide basic background mathematics for elementary school teachers.[29] The program was developed by an advisory group of mathematics specialists and educators, tried out in pilot school systems, and revised on the basis of findings. A study guide accompanies the course.

The mathematics content for the fifteen lessons is selected from the following areas:

[28] Henry Van Engen, "Patterns in Arithmetic" (available from Department of Education and Mathematics, University of Wisconsin, Madison, Wisconsin).

[29] Available on video tape or kinescope from M. Virginia Bigxy, Director, Center for Instructional Television, Eastern Educational Network, 238 Main Street, Cambridge, Massachusetts 02142.

Number and sets
Operations
Numerals
Algorithms
Integers
Arithmetic numbers
Decimal fractions
Rational numbers
Finite systems
Points in space
Numbers and points

The project was supported by the U.S. Office of Education on contract with the Northeastern Regional Instructional Television Library Project of the Eastern Educational Network.

"Foundations in Mathematics for Elementary Schools"[30] is an in-service program financed by the National Science Foundation, the Des Moines Public Schools, and the State College of Iowa. The purpose of the program is to help the teacher learn mathematics so that he can give his pupils more effective learning experiences in mathematics. The program is composed of thirteen thirty-minute lessons presented by kinescopes or video tapes, with notes for each lesson relating to the content of the telecast and serving to introduce and augment the telecast.

The authors recommend that the services of a consultant be obtained for two or three visits throughout the series to extend the lessons, clarify ideas, and discuss implications and methods of teaching.

The School Mathematics Study Group has prepared a text for in-service education of elementary school teachers. This text, ***A Brief Course in Mathematics for Elementary School Teachers,*** is designed specifically for teachers with little or no previous course work in mathematics.[31] Concepts that are basic to understanding the structure of our mathematical system are emphasized. It presents the minimum mathematical background that SMSG considers essential for an elementary school teacher, regardless of the program followed in the school.

The volume contains units with the following titles:

Numbers and Their Names
Place Value

[30] E. Glenadine Gibb and Augusta L. Schurrer, "Foundations in Mathematics for Elementary Schools," 1963. (Available from Director of Field Studies, State College of Iowa, Cedar Falls, Iowa.)

[31] Max S. Bell *et al., A Brief Course in Mathematics for Elementary School Teachers* (School Mathematics Study Group, "Studies in Mathematics," Vol. IX [Stanford, Calif.: Leland Stanford Junior University, 1963]). Distributed by A. C. Vromans, 367 South Pasadena Avenue, Pasadena, California.

What Are Numbers?
The Number Line
Addition and Its Properties
Addition and Subtraction
Addition and Subtraction Techniques
Multiplication
Division
Techniques of Multiplication and Division
Division as an Operation and the Division Algorism
Geometry—Points, Lines, and Planes
Geometry—Closed Curves, Non-metric Ideas
Geometry—Metric Properties of Figures
Linear and Angular Measures
Factors and Primes
Physical Models for Rational Numbers
The Rational Numbers
Addition and Subtraction of Fractions
Multiplication of Fractions
Multiplication and Division of Fractions
Decimals as Rational Numbers
Ratio
Congruence and Similarity of Plane Figures
Common Figures of Plane Geometry
Common Figures of Solid Geometry
Area
Volume
Extending the Number System Again
The Real Numbers
Coordinates

Each unit has class exercises. Answers are included.

Thirty half-hour films have been prepared to supplement the textbook.[32] One is to be used with each of the topics developed in the text. The first sixteen films provide a background for the mathematics usually taught in Grades K–3. The remainder extend the background to the mathematics normally taught in Grades 4–6.

Supplementary Modern Mathematics is an in-service course for teachers developed by the Madison Project.[33] The course consists of approximately fifteen lessons and film excerpts of around ninety minutes each selected from Madison Project films. Financial support for the project is

[32] Available from Modern Learning Aids, 3 East 54th Street, New York, N.Y.
[33] Robert B. Davis *et al., Supplementary Modern Mathematics for Grades 1 Through 9, In-Service Course for Teachers* (Reading, Mass.: Addison-Wesley Publishing Co., 1965).

provided by the National Science Foundation, the Division of Cooperative Research at the Office of Education, and other agencies. The purposes of the in-service program are (1) to help teachers become familiar with some new ideas to broaden the content of the elementary school program—ideas such as rectangular coordinates, the arithmetic of signed numbers, the use of variables, and the concept of mathematical function; and (2) to suggest ways in which creative learning, which is just as valid for mathematics as for any other subject area, can become part of the school program in mathematics.

As teachers read the material prepared for each lesson they are encouraged to try the activities themselves before they view the film excerpt, which shows a teacher and children in a classroom working with the same activity.

The lessons are planned for one or more school staffs to use under the direction of a discussion leader or a steering committee, who should be selected from the staff well in advance of the session to allow for adequate planning. Sometimes discussion questions for the lesson are suggested. Sometimes topics for discussion are left to the discretion of leader and staff. The text, *Discovery in Mathematics*, may also be used as a part of the course.[34] A reference book provides answers to questions and problems; significant points on the film excerpts; explanations of mathematical ideas, including diagrams to illustrate activities; discussions of children's responses; discussion of the value of guidance as opposed to the discovery method; and creative teaching techniques and ways of fostering enthusiasm and interest.

A two-week **Chicago workshop** combined closed-circuit television for presenting lectures and discussions, demonstrations with children, and films in small discussion groups under the leadership of a team of forty group leaders who had themselves studied Madison Project materials. Many of the 320 teachers in the workshop returned to their own schools and conducted similar courses for their coworkers.

The National Council of Teachers of Mathematics has prepared a series of ten in-service teacher-education films (16 mm., color, sound), **"Films in Mathematics for Elementary School Teachers,"** each approximately one-half hour long. They were produced by the NCTM with financial support from the National Science Foundation. These films are intended to acquaint teachers with the content of modern elementary school mathematics courses and to exemplify pedagogical techniques that have proven effective with such content. Text materials to accompany the films are distributed by the NCTM through its Washington office.

The film series is made up of the following ten films:

[34] Robert B. Davis, *Discovery in Mathematics—A Text for Teachers* (Reading, Mass.: Addison-Wesley Publishing Co., 1964).

1. Beginning Number Concepts
2. Development of Our Decimal Numeration System
3. Addition and Its Properties
4. Multiplication and Its Properties
5. Subtraction
6. Division
7. Addition and Subtraction Algorithms
8. Multiplication Algorithms and the Distributive Property
9. Division Algorithms
10. The Whole-Number System—Key Ideas

A text and a series of ninety half-hour films make up the in-service program ***Modern Mathematics for the Elementary Teacher,*** developed by Professor Leslie A. Dwight.[35] The project received support from the National Science Foundation and from participating local schools, which contributed $5.00 for each elementary school teacher in the system.

The two filmed lectures, presented each week by television, were repeated so that teachers had the opportunity to view each lesson twice.

The text provides study material on the concepts of mathematics to be studied before each filmed lecture is viewed. It combines content and method to achieve the following purposes:

1. To provide an opportunity for in-service and preservice elementary school teachers to acquire the mathematical background necessary to—
   a) Present with confidence and understanding the mathematical concepts recommended by national study groups for the elementary school curriculum today and in the near future.
   b) Develop within the pupils the spirit of modern mathematics.
2. To assist elementary teachers in their efforts to make mathematics come alive so that pupils may regard mathematics as a live and growing creation of the human mind
3. To provide a basis for further study of mathematics

The text materials have been tried out for several summers with groups of elementary teachers, supervisors, and principals, and with demonstration classes of elementary school pupils, as a part of the NSF summer institute program at Southeastern State College.

The following outline gives the content of the text and shows how films and text materials complement each other.

[35] Information on this program, available to other regions since 1964, may be obtained from Professor Leslie A. Dwight, Department of Mathematics, Southeastern State College, Durant, Oklahoma.

Chapter number	*Film number*	*Chapter content*
1	1	There is need to revise the school mathematics program.
2	2	The chapter discusses the objectives of teaching arithmetic, which influence the selection of content and justify techniques of presenting concepts.
3	3–14	This chapter presents a study of sets, set language, and operations on sets. Sets and set operations serve as a foundation for the development of all the basic mathematical concepts.
4	15–18	Sets provide a foundation for the development of the concept of number (cardinal) as an abstraction associated with sets.
5	19–31	The chapter develops the nature of numeration systems, including the topics of different bases, reading numerals, and the Roman system of notation.
6	32–48	Operations (binary) on numbers (cardinal) are introduced as abstractions from operations on sets. Fundamental laws (properties) of operations on sets are given. Applications of the laws are frequently included.
7	49–58	The law of inverse numbers and order laws are introduced, completing the basic structure of the rational-number system.
8	59, 60	Subtraction and division operations are introduced as inverse operations of addition and multiplication. They are also introduced through activities with sets.
9	61–90	Techniques for presenting the skills of the fundamental processes through concrete settings and the fundamental laws; study of fractions, ratio, and proportions; and decimals, percent, problem solving, and geometry, are included.

This program seeks to unify the many isolated topics in traditional mathematics by presenting the basic structure of the rational-number system.

Leadership Training Workshops

Leadership training institutes have been held in several states for the purpose of training instructors for workshops for elementary school teachers. These have been financed by the National Science Foundation, the National Council of Teachers of Mathematics, state departments of education, and local school systems with the cooperation of other agencies. The following are several examples of these programs:

The mathematics supervisor for the state of **Washington** has provided leadership training for selected teachers of junior or senior high schools to prepare them as in-service instructors of elementary school teachers. Candidates have been brought together for one or two weeks' orientation to the

general elementary school program and to the mathematics of the elementary school. Often teachers who have attended NSF institutes or recent courses on new approach to content and method are likely recruits. They are selected because they know mathematics and understand the needs of elementary school teachers.

The many instructors needed to implement the expanding program of in-service education in the state of **New York** participate in a leadership conference before serving as instructors for groups of teachers. Participants in the training program are paid $25.00 per day. An instructor's manual, prepared by the first group of regional instructors in the New York Regional Program for Elementary Teachers, is used as a basis for the instruction of leadership personnel. The original manual, developed at a summer workshop, undergoes continuous revision as it is used. It includes these chapters:

Number, Numeral, and Numeration
Principles Necessary for the Structure of the Number System
Logical Development of the Number System
Problem Solving
Operations with Whole Numbers
Fractions as Numbers and as Relationships
Measurement
Geometry

The National Council of Teachers of Mathematics supported a pilot leadership training project in the state of **Idaho.** Selected prospective instructors for in-service programs for elementary school teachers were drawn mainly from the group of secondary mathematics teachers who had attended NSF institutes or academic-year programs. A few principals and superintendents were also included in the group. The selected candidates were brought together for a two-week orientation workshop designed to acquaint them with the elementary school program in general, with the mathematics of the elementary school, and with the materials available for in-service work with elementary school teachers.

The training program included—

1. A study of elementary school programs developed in demonstration projects, such as text materials, teacher's manuals, and films.
2. Discussions with special teachers who had successful experience in teaching mathematics at the elementary school level.
3. Setting up specific plans for in-service education of elementary school teachers in their respective school districts.

During the first year of the program, the twenty-four persons who received training became mathematics consultants for the school districts, with respon-

sibility for in-service work with elementary school teachers in the district.

The department of education of the **University of Wisconsin** is now offering a doctoral program in education with concentration in elementary school mathematics education. Under this program assistantships provide experience in supervision of undergraduate students, in work with in-service education programs, and in various research projects.

For several years the mathematics education department, **Syracuse University,** has provided a program encouraging selected elementary teachers to develop competence in mathematics sufficient to enable them to serve as master teachers or resource persons for mathematics within the school. Professor Vincent Glennon states two reasons for the program: first, that the principal can no longer be responsible for supervision of all subjects in a total instructional program within the elementary school; and second, that every school needs one teacher who knows mathematics very well.

Schools in the vicinity of Syracuse having specialist teachers in their schools have found it helpful for the specialist to have some released time for demonstration teaching, assisting teachers with materials, meetings with the staff, and conferences with teachers by grade or individually. The program is an attempt to have someone available at all times to answer teachers' questions about mathematics and to provide leadership for the school within this subject area.

A specific illustration of this program is the project recently initiated by the **Schenectady public school system.** The three-year project will operate under a grant from the New York State Department of Education's locally organized in-service program. The project is being coordinated by Syracuse University under the direction of Professors Vincent Glennon and John Wilson with the assistance of other staff members. During the first year, the 250 teachers involved in the project will take the course entitled "Mathematics for the Elementary School Teacher" offered by the university. During the second and third years, the university will offer training in depth by means of a planned sequence of courses for one person from each elementary school. The project will be designed to prepare these selected teachers for service as teacher-leaders who will assist other teachers in improving the mathematics program within their respective schools.

Each summer the **National Science Foundation** sponsors a number of mathematics institutes for elementary school, training leadership personnel for mathematics programs. The institutes, which are from five to eight weeks long, are held in colleges and universities over the country. Key teachers (those having some responsibility for leadership), supervisors, and principals concerned with instruction in mathematics are eligible to participate. The foundation grant to each institution covers the cost of tuition, fees,

stipends, and allowances. Participants are selected by the institute. Information on institutes available at any given time may be obtained from NSF, but application forms must be requested from the host college or university.

An NSF institute with a unique follow-up program, conducted cooperatively by the mathematics and education departments of the **University of Vermont,** offered a six-week mathematics program for elementary school teachers, supervisors, and administrators.[36]

The first part of the program was the summer study of the basic concepts of elementary school mathematics, which included the following topics:

The real-number system and its subsystems
Sets, logic, and systems of numeration
Properties of numbers, relations, and operations
An introduction to algebra
Intuitive foundations of geometry

The second part of the program took place in the local system, where each workshop participant conducted an in-service course for teachers in the district. While the program in each locality varied to some extent to meet the needs or demands of the local system, all emphasized the mathematics content from the summer institute. Other study activities included an analysis of modern textbooks, a review of films on new mathematics, and a discussion of effective classroom procedures.

A survey revealed that more than 800 teachers teaching over 20,000 children participated in the local follow-up in-service courses.

San Jose State College, San Jose, California, offers an Academic-Year Institute program sponsored by NSF to prepare supervisors and selected junior high school teachers to conduct in-service education programs for elementary school teachers. One requirement for acceptance into the program is evidence of a need for an in-service program in the applicant's school district and assurance that arrangements will be made for such programs following the training program. Applicants must have a minimum of three years' teaching experience and meet the mathematics requirements of the college for acceptance.

A course entitled "Observation and Practice in Teaching of Contemporary Mathematics Programs" provides opportunities for participants to observe modern classes, examine and study materials from current programs, engage in seminar discussions of problems related to teaching, view films of modern mathematics classes, and receive guidance in developing an in-service course for use in their local systems.

[36] Joseph A. Izzo and Ruth Kelley Izzo, "Re-education in Mathematics for Elementary School Personnel: In-Service Programs—One Way to Solve the Problem," *The Arithmetic Teacher,* XI (October 1964), 413–17.

III

The Challenge Ahead

THE TEACHERS in our elementary schools have at least three general areas within which they need to be competent to conduct the inquiries related to their teaching tasks. One of these is certainly the subject-matter fields from which the content of the elementary curriculum is largely drawn. A second is the area of child growth and development. A third is the whole field of learning and instruction."[37]

The fact that mathematics is taught in every grade of the elementary school lends evidence to its importance and emphasizes the need for teachers to be well prepared in the area. The total task of teaching as described by Rehage in the quotation above expresses concern, not only for the aspect of teachers' knowledge of subject matter, but also for the psychological and pedagogical aspects. For instance, teachers must be concerned with the selection and adaptation of material in consideration of the abilities of children, with methods of presentation, and with evaluation of the effectiveness of both teaching and learning.

GOALS AND STANDARDS

When the opportunities for in-service education are extended to include most of the elementary school teachers in the nation, we will be able to achieve certain well-conceived goals and to meet some needs that still exist. Hopefully, we will be able to—

1. Help teachers see mathematics as the imaginative, creative study of patterns leading to generalized, abstract ideas, not as mechanical computation.
2. Give more consideration to the way in which the content of modern mathematics and older programs may be brought together to avoid gaps, overemphasis of some areas, and underemphasis of others.

[37] Kenneth J. Rehage, "Teachers for Emerging Elementary Schools," *Frontiers of Elementary Education*, VI (Syracuse, N.Y.: Syracuse University Press, 1959), 13.

3. Achieve an effective teaching-learning sequence that is continuous and well integrated throughout the elementary years and with the secondary program.
4. Reorganize the total program so that modern topics such as sets, geometry, and structure of mathematics truly become unifying ideas that form the fundamental webwork of the total curriculum and do not merely appear as units interspersed here and there.
5. Determine the topics or parts of topics that can be most effectively taught by the discovery method.
6. Continue to participate in experimental programs designed to explore better curricula and more promising methods.
7. Evaluate more thoroughly what is being done and determine changes indicated by the evaluation.
8. Disseminate information effectively so that the lag between knowledge and classroom practice will be lessened.
9. Study the extent to which the concepts of mathematics and the teaching procedures that appear of worth are also applicable to other subject areas, and the extent to which the correlation between mathematics and other subject areas may be explored to save time in learning, to increase understanding of concepts, and to illustrate the applications of content.

For minimal competence, elementary school teachers should achieve the the recommendations of the Committee on the Undergraduate Program in Mathematics (CUPM) of the Mathematical Association of America. CUPM assumes that candidates for elementary school teaching have competence in arithmetic and at least two years of college-preparatory mathematics, including a year of algebra and a year of geometry. On this basis CUPM recommends the following for all elementary school teachers (who are classed as Level I):

1. A two-course sequence devoted to the structure of the real-number system and its subsystems
2. A course devoted to the basic concepts of algebra
3. A course in informal geometry [38]

In order to make the Level I recommendations more explicit, the Panel on Teacher Training of CUPM developed detailed outlines for courses in

[38] *Ten Conferences on the Training of Teachers of Elementary School Mathematics*, Report of the Committee on the Undergraduate Program in Mathematics (Buffalo, N.Y.: Mathematical Association of America, 1963), p. 18.

number systems, algebra, and geometry. This outline is now being used as a basis for developing text materials.

Topic outlines for the three courses are given below.[39]

Course I—Structure of the Number System

- Introduction
- The Language and Nature of Deductive Reasoning
- Elements of Set Theory
- The Whole Numbers
- Numeration Systems
- Additional Operations
- Positive Rational Numbers
- Introduction of Negative Rational Numbers
- Elementary Number Theory
- Decimals and the Real Numbers

Course II—Algebra

- Introduction
- Properties of Real Numbers
- Linear Equations and Inequalities
- Quadratic Equations and Inequalities
- Systems of Linear Equations and Inequalities
- Modular Arithmetic
- Complex Numbers
- Polynomials
- Algebraic Structures
- Functions

Course III—Geometry

- Introduction
- Experimental and Informal Geometry
- Sets, Points, Lines, Planes, Space
- Elementary Theorems and Proofs
- Congruence and Measurement of Segments and Angles
- Congruence of Triangles and Geometric Construction
- Parallels and Parallelograms
- Space Figures
- Similarity and Trigonometry
- Area and Volume Measurement

[39] *Course Guides for the Training of Teachers of Elementary School Mathematics* (available from Committee on the Undergraduate Program in Mathematics, P. O. Box 1024, Berkeley 1, California).

Measurements Related to Circles
Elements of Spherical Geometry
Plane Coordinate Geometry
The Postulational Method

CUPM considers its recommendation to be the minimum requirement for competent teaching of elementary school mathematics. At the same time there is a recognition that successful teaching involves more than content: "It is assumed that topics will contain a judicious mixture of motivation, theory, and application. A purely abstract course for teachers would be madness, but a course in calculation with no theory would not be mathematics."[40]

CUPM strongly recommends that about one in four or five elementary school teachers have training beyond the Level I courses in order to strengthen school faculties in the area of mathematics and to prepare themselves for positions of leadership as special teachers of mathematics or as consultants.

For teachers who are preparing to serve in these or similar capacities, study in greater depth is needed. The recommendation of CUPM is that these teachers meet their recommendation for junior high school teachers (classed as Level II by CUPM). Courses would include a study of abstract algebra, of geometry, and of probability and some work in the language of logic and sets. Some colleges and universities have considered setting up an academic-year program for teachers who wish to prepare for positions of leadership in elementary school mathematics.

HOW SCHOOL SYSTEMS CAN ENCOURAGE IN-SERVICE EDUCATION

As all school personnel become convinced of the absolute necessity for continuing education, the ability to take advantage of those features and practices that will facilitate in-service programs may mean the difference between success and failure. In reality, in-service education begins with the first day of teaching and continues throughout the teacher's career. Some noteworthy efforts that encourage participation in in-service programs are described in this section.

School systems and colleges and universities are recognizing the need for cooperation during the first year or two of the teacher's on-the-job experience. As a result many institutions continue to maintain close contact with the teacher by visits, conferences, back-to-college seminars, correspondence, and other means.

Some school systems are extending the school year for teachers well beyond

[40] *Recommendations for the Training of Teachers of Mathematics: A Summary* (Buffalo, N.Y.: Mathematical Association of America, 1961), p. 8.

the number of days of pupil attendance in order to provide for continuing education within the paid working day of the teacher.

The sabbatical leave is being recognized by some school systems as one way of meeting the need for professional growth of selected elementary school teachers.

Every elementary school should have at least one teacher who is a member of the National Council of Teachers of Mathematics so that all teachers within the school may be informed of the activities of the council that are of interest to elementary school teachers. Certainly every school should have an institutional membership to NCTM, making the *Arithmetic Teacher* magazine available to every member of the teaching staff, and should add to the professional library of the school those yearbooks of the Council that deal with the mathematics program of the elementary school.

One state offers grants-in-aid for summer work at participating colleges and universities. Stipends of $300 for a six-week summer session and $450 for nine weeks are allowed. To be eligible, recipients must have a bachelor's degree, achieve a certain score on the National Teachers' Examination, and agree to teach one year in the state following the summer of study.

Some school systems provide released time for teachers during the year for professional study locally. Plans of one-half day once a month or two hours twice a month have been successful in certain systems. In some cases the school system gives an hour of time and the teachers give an hour, so that, to attend a two-hour in-service class, the classroom teacher need stay only one hour beyond the ordinary closing time.

Some local systems pay selected teachers by the day to develop curriculum materials during the summer months. Enrichment units or materials for fusing the modern mathematics with regular programs have been developed in this way.

Some school systems pay all expenses for selected teachers to attend an institute or summer-school program with the understanding that these teachers will assist in conducting noncredit in-service programs for elementary school teachers during the following year.

Teachers who are to participate in a local in-service program are often requested to indicate which of several time schedules would be most convenient for the group concerned. Each of the following plans has been used successfully by some school systems:

1. Weekly two-hour sessions—4:00 to 6:00 or 7:30 to 9:30
2. Dinner meetings on alternate weeks with a two-hour session before and another two-hour session after dinner
3. Saturday morning sessions of three hours each

4. After-school sessions from one to two hours long

   (While teaching and studying simultaneously is taxing on the teacher's energy and time, the combination provides good motivation for learning and for effecting change in classroom teaching.)

The need for long-range in-service programs that are projected three or more years into the future is being recognized by some school systems as essential in order to meet the different backgrounds and different competence of teachers in the system.

School systems have found that a coordinating committee for mathematics in-service education with representatives of Grades K–12 is helpful in making long-range in-service plans.

One school system allocated 8 percent of the normal school week to in-service education.[41]

Some school systems have organized in-service work for teachers of primary grades (1, 2, and 3) and of intermediate grades (4, 5, and 6). In some instances where the in-service course was designed to implement a new textbook series, the organization has been by single grades.

THE EVALUATION OF IN-SERVICE PROGRAMS

The real effectiveness of an in-service program is measured by changes in teaching practices and by improvement in the learning of the children. Evaluation of any program is dependent upon whether the measuring instruments used evaluate the concepts of the program. While such instruments are being devised, tried out, and validated, there are many informal yet promising practices yielding cumulative evidence to local school systems and state departments of education of the effectiveness of in-service programs. Among them are the following:

Records are kept on the number of classes participating in in-service programs, the organization of the classes, the number of participants per class, and so on.

Participants make informal evaluations of the strengths and weaknesses of each session and of the course as a whole, and evidence is gathered on the extent to which the course was helpful in producing changes in the classroom.

Surveys are made to determine if teachers who have participated in a modern mathematics in-service course are more likely to select textbooks reflecting a modern approach than teachers who have not experienced such a program.

Attitude scales are used before and after a course to provide some evi-

[41] George L. Roehr, "A Report of In-Service Education," *School Mathematics Study Group Newsletter*, No. 15 (Stanford, Calif.: Stanford University School of Education, 1963).

dence of changes in attitudes toward mathematics produced by the course. Similar scales provide some evidence on changes in attitudes of children in upper elementary grades toward mathematics.

Traditional tests are used to determine how children in modern programs compare with others of the same grade in abilities and skills covered by these tests.

Informal tests are developed to determine children's understanding of the newer concepts.

The achievement of children who are in a new program for the first time is compared with the achievement of children who have been in a new program for one or more years.

High school requirements are evaluated to be sure that all college-bound students take college preparatory mathematics, because it is from this group that future teachers will come.

One effort to evaluate an in-service program utilizes a team-teaching approach, including TV, lectures, question-discussion sessions, and written materials.[42]

A teaching team from the University of Texas, in cooperation with the Austin Public Schools, presented a series of five weekly ninety-minute sessions emphasizing both content and method. The topics for the five programs were these:

1. Number and Numeral
2. Base and Place
3. Structure of the Number System and Its Relation to the Four Fundamental Processes
4. Some Laws of Arithmetic
5. A Model Theory in Mathematics

The 252 participating teachers and administrators represented a wide range in teaching experience and mathematics background. Results showed that 71 percent of the participants found the series useful. Eighty percent indicated that they had changed classroom practice to include newer materials. These teachers found TV and lectures highest in effectiveness, question-discussion sessions next highest, and written materials next. Teachers with four or more years' experience in teaching rated the series higher than those with less experience. Lectures were ranked higher by teachers with four or more years' experience. Those with less than four years' experience ranked TV and question-discussion sessions higher.

[42] W. Robert Houston, Claude C. Boyd, and M. Vere DeVault, "An In-Service Mathematics Education Program for Intermediate Grade Teachers," *The Arithmetic Teacher*, VIII (February 1961), 65–68.

The authors see a need for individualizing in-service courses in terms of the interest, capabilities, and goals of teachers. Their findings indicate that in-service programs should be different for beginning and for experienced teachers—different in content, in methods of approach, and in the use of media.

The authors suggest that case studies, classroom observations, pupil-interview techniques, and experimental studies involving pretesting and posttesting of teachers and pupils be utilized in the evaluation of in-service programs.

FUTURE GOALS FOR SCHOOL MATHEMATICS

In 1963 a conference of twenty-five mathematicians, including specialists in both pure and applied mathematics, was held to consider long-range goals of the mathematics curriculum of the schools.[43] Recommendations for the elementary grades included—

The use of the number line (*a*) to introduce the child early in his school life to the entire system of real numbers and (*b*) to keep him aware from the beginning that such numbers as negatives and fractions exist and that these numbers can be added and multiplied just like whole numbers.

An early extension of the number line to Cartesian coordinates or cross-number lines.

Emphasis from the start on the close relationships between arithmetic, geometry, and informal algebra.

Highlighting of the properties of number systems and of operations with each subsystem of the real-number system.

Maintenance of a constant interplay between mathematics and its application to the physical world.

An introduction to some of the ideas of probability and statistics.

The inclusion of such topics as set and function as they contribute to the understanding of topics being studied.

Extensive use of zero as a number.

Introduction of the idea of inequality and the symbols of inequality.

An understanding of inverse operations.

Development of reasonable proficiency in computation through problems that catch student interest. The committee maintains that long pages of computations can destroy interest, particularly for brighter children.

[43] *Goals for School Mathematics*, a report of the Cambridge Conference on School Mathematics, sponsored by Educational Services Incorporated (Boston, Mass: Houghton Mifflin Co., 1963).

The committee acknowledges that the goals are conjectures to be tested when experimental programs of the future are designed around them and research techniques are devised to test their worth. Important factors in this procedure will be the following questions:

Can the goals be broken down into simpler concepts and fitted into a sequence that can be handled by elementary school children?

Can children grasp the relationships that are basic to the goals?

For what abilities of children can these goals be accomplished? Is the direction set by them appropriate for all children?

What can be done within the limitation of time, and with due consideration for all other important areas of the elementary school curriculum?

Certainly such questions are a challenge to the combined efforts of educators and mathematicians if workable answers are to be found.

Summary Statement

A VAST majority of teachers at the elementary school level are responsible for mathematics instruction for children in their classes. This makes the task of preparing teachers at this level ten times as large as that at the secondary level, where only one of several teachers has direct responsibility for the mathematics program.

There are many unsolved problems that relate to the in-service education of elementary school teachers. Among them are the following:

What effect will the improved mathematics courses for prospective elementary school teachers have on the future in-service offerings for these same teachers?

To what extent can the methods and content of mathematics be applied to other subject areas? How can discovery and creative teaching become key methods of teaching all subjects? Does the concept of sets have applications to the organization of content in science, social studies, and the language arts?

How can the mathematics program be redesigned to provide more appropriate programs for able children, for slow children, and for those with other types of special need?

What organizational procedures are the most promising from the standpoint of children's learning and their greatest total growth as individuals?

How are changes accomplished in teaching and learning?

What mathematical learning is most closely related to the understanding of the mathematics of contemporary elementary school programs?

Who should be involved in the development of curriculum guides and similar materials for teachers?

What types of in-service courses (content, combined content and method, or other types) are most likely to produce changed behavior in teachers and in children?

What mathematics content is most basic for effective teaching by elementary school teachers?

While scant evidence from research is available on some of these prob-

lems, in most cases it is necessary to operate on information gathered from experience that has been studied and analyzed on the basis of informed and reasoned judgment.

The need for in-service education for all teachers, even for those who have recently had modern mathematics courses in college, is pointed up in a study of the 1963 graduates who were beginning their teaching in six suburban districts of Philadelphia.[44] While the teachers in the study claimed they had taken college courses in modern mathematics, results indicated only 36 percent mastery as the median score on items selected from the following topics: the language of sets, numeration systems, the language of geometry, equalities and inequalities, use of the number line, use of ratio, and application of mathematical laws.

Further evidence to support the concept of mathematics in-service education as a continuous process is found in a study of academic and other background factors that are pertinent to the teacher's understanding of mathematics.[45] Significant relationship was found between mathematical understanding and each of the following: scholastic aptitude scores, college scholarship, high school and college grades, students' favorable self-appraisals of mathematical understanding, and study of an arithmetic methods course. No significant relationship was found between the teacher's mathematical understanding and any of the following: number of years mathematics was studied in high school, number of college mathematics courses taken, number of years of teaching experience, experiencing mathematics that emphasized practical life situations, or experiencing mathematics that emphasized meaningful learning.

It may be inferred from such evidence that the problem of teachers' gaining understanding involves factors other than content mathematics courses and experience. At least these findings tend to indicate that the ability of the prospective teacher and perhaps the quality of courses and experience and the relationship to the profession of teaching are factors of equal importance. The recent trend toward designing mathematics courses particularly for teachers shows promise of being more fruitful than courses of the past.

There is a difference of opinion about the extent to which teachers themselves should be engaged in the development of curriculum materials such as guidelines, guides, and manuals. Holders of one point of view maintain that teachers lack the knowledge needed for curriculum development—that only expert mathematicians are qualified to determine what should be taught. Holders of the other point of view maintain that users of materials should

[44] Ruth Melson, "How Well Are Colleges Preparing Teachers for Modern Mathematics?" *The Arithmetic Teacher*, XII (January 1965), 51–53.

[45] Emma C. Carroll, "A Study of the Mathematical Understanding Possessed by Undergraduate Students Majoring in Elementary Education" (doctoral dissertation, Wayne State University, 1961). *Dissertation Abstracts*, XXII (August 1961), 494.

have the opportunity to participate in the development of them—that consultants, to be effective, should work *with*, not merely *for*, those they would help. From this point of view curriculum development would take place within a single school or within a school system with as many teachers as possible participating. It seems reasonable to assume that mathematicians and school personnel have strengths that are different but complement each other.

Mathematics in-service education should be a concept of continuous study. Because of the constantly changing curriculum and the neglect or absence of mathematics in preservice education, there will always be a next level for elementary school teachers and for others responsible for the mathematics education of children. It is to assist in the realization of this goal that the many varied suggestions and activities described in this bulletin have been presented.

Appendices

APPENDIX A: BIBLIOGRAPHY

NOTE.—By permission of J. Fred Weaver, the following entries, with three exceptions, are drawn from two bibliographies that appeared in his valuable department "Focal Point," in the *Arithmetic Teacher* (January 1965 and December 1965). The additional entries are for the books by Dutton, Flournoy, and Heimer and Newman.

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APPENDIX B: SOURCES OF MATHEMATICS INFORMATION

OFFICIAL TITLES AND ADDRESSES OF STATE MATHEMATICS SPECIALISTS

Mathematics Consultant
State Department of Education
Montgomery, **Alabama** 36104

Education Supervisor
Science & Mathematics
State Department of Education
Juneau, **Alaska** 99801

Consultant in Secondary
Education (NDEA)
State Department of Education
Sacramento, **California** 95814

Consultant, Science & Mathematics
State Department of Education
Denver, **Colorado** 80203

Consultant, Mathematics Education
State Department of Education
Hartford, **Connecticut** 06115

State Supervisor of Mathematics
State Department of Public Instruction
Dover, **Delaware** 19901

Supervisor, Mathematics Education
State Department of Education
Tallahassee, **Florida** 32304

Coordinator of Mathematics Education
State Department of Education
Atlanta, **Georgia** 30303

Program Specialist, Mathematics
State Department of Education
Honolulu, **Hawaii** 96813

Science & Mathematics Supervisor
State Department of Education
Boise, **Idaho** 83701

Mathematics Consultant
Office of the Superintendent of
Public Instruction
Springfield, **Illinois** 62706

Supervisor of Mathematics & Science
State Department of Public Instruction
Indianapolis, **Indiana** 46206

Regional Consultant
State Department of Public Instruction
Des Moines, **Iowa** 50319

Consultant in Science & Mathematics
State Department of Public Instruction
Topeka, **Kansas** 66601

Supervisor of Mathematics, Part-Time
State Department of Education
Baton Rouge, **Louisiana** 70804

Supervisor of Mathematics
State Department of Education
Augusta, **Maine** 04301

Supervisor of Science & Mathematics
State Department of Education
Boston, **Massachusetts** 02109

Mathematics Consultant
State Department of Education
St. Paul, **Minnesota** 55101

Supervisor of Mathematics
State Department of Education
Jackson, **Mississippi** 39205

Mathematics Supervisor
State Department of Education
Jefferson City, **Missouri** 65101

Science & Mathematics Supervisor
State Department of Public Instruction
Helena, **Montana** 59601

Mathematics Consultant
State Department of Education
Carson City, **Nevada** 89701

Director of Mathematics Education
State Department of Education
Concord, **New Hampshire** 03302

Consultant, Mathematics & Science
State Department of Education
Trenton, **New Jersey** 08625

Director of Mathematics
State Department of Education
Santa Fe, **New Mexico** 87501

Supervisor, Mathematics Education
State Department of Education
Albany, **New York** 12224

Supervisor of Mathematics
State Department of Public Instruction
Raleigh, **North Carolina** 27602

Supervisor of Mathematics
State Department of Education
Columbus, **Ohio** 43216

Consultant in Mathematics
State Department of Education
Salem, **Oregon** 97301

Mathematics Specialist
State Department of Public Instruction
Harrisburg, **Pennsylvania** 17126

Supervisor, Mathematics
State Department of Education
Providence, **Rhode Island** 02904

Supervisor for Mathematics
State Department of Education
Columbia, **South Carolina** 29202

Supervisor of Mathematics
State Department of Education
Nashville, **Tennessee** 37209

Consultant in Mathematics
Texas Education Agency
Austin, **Texas** 78711

Supervisor of Mathematics
State Department of Public Instruction
Salt Lake City, **Utah** 84114

Consultant in Science & Mathematics
State Department of Education
Montpelier, **Vermont** 05601

Supervisor of Mathematics
State Board of Education
Richmond, **Virginia** 23219

Supervisor, Mathematics Program
Office of State Superintendent of
Public Instruction and
State Board of Education
Olympia, **Washington** 98501

Mathematics Specialist
State Department of Education
Charleston, **West Virginia** 25305

Supervisor of Mathematics
State Department of Public Instruction
Madison, **Wisconsin** 53702

Supervising Director of Mathematics
Public Schools of the District of Columbia
Washington, **D.C.** 20009

Director, Mathematics Program
Department of Education
Hato Rey, **Puerto Rico** 00900

Addresses to Be Used When States and Territories Have No Specialist Supervisor for Mathematics

Director of Secondary Education
State Department of Public Instruction
Phoenix, **Arizona** 85007

Coordinator, NDEA
State Department of Education
Frankfort, **Kentucky** 40601

Consultant
Title III, NDEA
State Department of Public Instruction
Lansing, **Michigan** 48901

Administrator
Title III, NDEA
State Department of Education
Lincoln, **Nebraska** 68509

Director of State and Federal Relations
State Department of Public Instruction
Bismarck, **North Dakota** 58501

Administrator, Defense Education
State Department of Education
Oklahoma City, **Oklahoma** 73101

Secondary School Supervisor
State Department of Public Instruction
Pierre, **South Dakota** 57501

Superintendent of Public Instruction
State Department of Education
Cheyenne, **Wyoming** 82001

Consultant for Title III
Department of Education
Agana, **Guam** 96910

Coordinator of Curriculum
Division of Schools
Balboa Heights
Canal Zone, **Panama**

Director of Education
Department of Education
Pago Pago, American **Samoa** 96920

Commissioner of Education
Department of Education
Charlotte Amalie, St. Thomas
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